$$h_{1,2} = \int_{t_1}^{t_2} V'dt \qquad (17)$$

$$= \int_0^{\pi} \frac{r}{Vc} V'd\theta = \pi r \frac{V'}{Vc}$$

since the bird is rising with constant velocity V′. On this leg of the loop the bird has gained both kinetic and potential energy.

On the circuit 2 to 1, that is, from downwind to upwind, the bird performs work in accelerating the air in the wind direction as it turns. This work is exactly $w_{2,1} = -2MVcW'$, the (—) sign denoting work done by the bird. Hence on making a complete circuit the net change in the kinetic energy of the bird is zero, and no energy has been extracted from the horizontal air motion. Indeed we see that it is not possible to gain any net energy from the horizontal motion.

The apparent loss of altitude on the downwind arc of a soaring bird is therefore illusory; the appearance is due to the fact that the *distance* rate of gain of altitude on the upwind leg is usually much larger than that of the downwind leg.

The flight path in thermal soaring. — The path relative to earth as traced out by a craft in equilibrium flight in a thermal shell can be expressed in precise mathematical form. This is extremely important since it will be shown that such a path coincides perfectly with the experimentally determined paths of soaring birds and hence offers confirmation of the validity of the theory presented. The inference is that we have learned one of the major secrets of soaring birds, with all its consequences. Conversely, with a few simple measurements the theory allows us to gain considerable information about the birds.

Mathematically, the flight path in space has the general nature of an inclined helix, but is not a true helix. The inclination of the axis depends on the relative values of V′ and W′, the thermal rising velocity and the wind speed. Its value is V′/W′. The axis is clearly the locus of the point of intersection of the vertical axis of the thermal shell with the η plane in which the craft is circling. The projection of this path on the earth's surface (i.e. the path relative to a ground observer directly under the craft) is a simple trochoid. Geometrically, a trochoid is the path traced in a plane by a point on the circumference of a circle as the circle rotates with constant angular velocity ω while simultaneously translating along a straight line with constant velocity. In the present case the circle corresponds to the path of the craft relative to the thermal core and hence has a radius r. The angular velocity ω is clearly Vc/r and the translation velocity is merely the speed W′ with which the wind carries the entire shell (and circle) along horizontally. If we define ϕ as the total angular displacement (in radians), of the line connecting the sailplane with the thermal axis, from some initial zero position the coordinates of the trochoid become

$$x = r\left(\frac{W'}{Vc}\phi - \sin\phi\right) \qquad (18)$$

$$y = r\left(\frac{W'}{Vc} - \cos\phi\right) \qquad (19)$$

The altitude coordinate z is determined from the relation

$$z = V't = \frac{V'}{Vc} r\phi \qquad (20)$$

since $\phi = \omega t = \frac{Vc}{r} t$. These three equations determine the flight path of a craft in equilibrium flight in a thermal shell, under the assumptions that the velocities V′ and W′ and the turn radius r, remain constant with altitude. Actually all three change gradually as the thermal shell rises. The equations therefore accurately describe the motion over finite segments of the path, the values of V′, W′, and r being assumed constant during any one segment.

The equations just developed accurately predict the observed flight paths of soaring birds. To obtain the x - y projection of the flight path we aim a large mirror skyward, directly beneath a soaring bird, and mark off on the mirror with ink the position of the bird at equal time intervals as indicated by a metronome. The time ticks are later connected by a smooth curve to give a flight path such as shown in figure 8A. It is of course possible to trace only a few such loops before the image passes off the mirror, the number depending upon the altitude and speed of the bird. However, such plots are easy to obtain where soaring birds are plentiful and Hankin* made many such records in India. On the flight path tracing we insert x - y axes such as shown in figure 8B. Then from the figure we can determine by direct measurement the value of the ratio x_1/r since the distance between envelopes of the flight path is 2r. The choice of x_1 is such that $\phi = 2\pi$. Then from equation (18) we can solve for W′/V′. Thus with the constants known we can plot the true trochoidal flight path and compare it with the experimental path, using equations (18) and (19). Figure 8B is an example of this method. Two loops, taken from the actual flight path of a Cheel, as recorded by Hankin in India in 1913, were

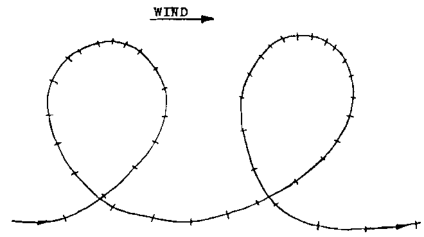

Fig. 8A. Flight path projections of a soaring bird.

* E. H. Hankin: *Animal Flight: A Record of Observations*, Iliffe and Sons, Ltd., London, 1913.

Fig. 8B. Flight path projections of a soaring bird.