position of stable equilibrium exactly the same vertical distance ($\eta$ +) above the core plane as it had been below it, provided of course the radius has not changed. By the same token, however, a slight downward displacement would cause the sailplane to sink out of the shell. Even while it was sinking relative to the core, it could actually gain altitude relative to earth if V′ were large enough.

The critical importance of the existence of a circulation within the thermal shell is evident. If this circulation did not exist a sailplane could only hope to enter near the top and to gain some degree of altitude as it rapidly sank through the shell. By virtue of the circulation, however, the craft can establish a permanent equilibrium and can remain within it for (theoretically) the practical life of the thermal. In order to gain maximum altitude the craft must attain equilibrium. Otherwise it will benefit from the upcurrent for only a short time. These facts point out the critical importance of the craft's ability not only to circle within the thermal but also to have a small enough sinking velocity that equilibrium can be attained. The difference in available energy depending on whether or not equilibrium is attained is enormous. The ability of soaring birds to attain equilibrium in even very small, weak thermals is the primary reason why they so hopelessly outclass sailplanes in the certainty of their flight. At present only the largest thermals are available to sailplanes. The smaller thermals are merely felt as "bumps" as the plane passes through them.

If the geometric vertical axis of the thermal were visible, it would appear to the pilot that he was continuously circling about this axis with a uniform velocity Vc ($= V \cos\theta$), where Vc may be called the circling velocity relative to the thermal. To the ground observer the motion will appear quite different and the velocity of the craft relative to earth will be constantly changing. As the craft moves with the wind it will speed up; as it moves against the wind its speed will decrease. The path generated will consist of continuous loops, each successive one being displaced in the wind direction. This type of flight path has been observed by students of birdflight for centuries and countless attempts have been made to construct theories based on the false premise that the bird is able to extract energy from the horizontal wind by virtue of the velocity fluctuations of the bird itself. The general idea is that the bird is able to use the velocity gained on the downwind leg to so increase its aerodynamic velocity on the upwind leg that considerable altitude can be gained on the upwind leg. The fallacy of course is that in order to turn into the wind, the bird must turn *relative to the air* and hence cannot possess any increased relative velocity unless he has done work on the air by expending internal energy or losing altitude.

It is also generally maintained that the bird must lose altitude on the downwind leg to furnish the energy for the increased velocity. I offer the following proof that it is not necessary to lose altitude on the downwind leg and that actually equal altitude gain occurs all around the loop in thermal soaring. Since the bird, or sailplane, maintains a constant circling velocity Vc relative to the air as it circles in the thermal shell, its absolute velocity when facing upwind is Vc — W′ and when facing downwind is Vc + W′ (figure 7). Then

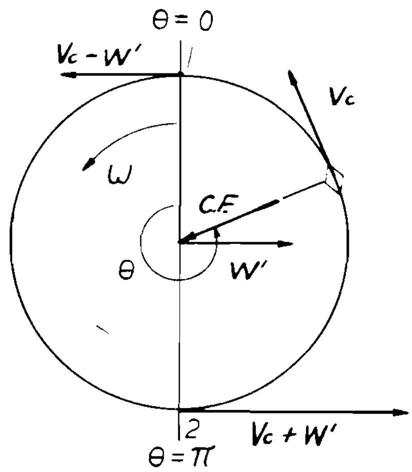

**Fig. 7. Vector diagram for the thermal circle.**

the change in kinetic energy between these two points 1 and 2 is by definition

$$\Delta KE_{1,2} = \tfrac{1}{2} M \left[ (Vc + W')^2 - (Vc - W')^2 \right] \quad (12)$$

where M($=$ W/g) is the mass of the aircraft. Expanding this we obtain

$$\Delta KE_{1,2} = 2MVcW'$$

This energy is equal to the work done by the centripetal force on the craft between points 1 and 2. Using the nomenclature of figure 7, the rate of working is

$$\frac{dw}{dt} = (\overrightarrow{Vc} + \overrightarrow{W'}) \cdot \overrightarrow{C.F.} \quad (13)$$

where $\overrightarrow{C.F.} = -M \frac{Vc^2}{r^2} \vec{r}$ is the centripetal force vector, $\overrightarrow{Vc}$ is the circling velocity vector, W′ is the wind vector, and w is the work. The dot denotes the scalar product of the vectors. Since $\overrightarrow{Vc} \cdot \overrightarrow{C.F.} = 0$ constantly ($\overrightarrow{Vc}$ and $\overrightarrow{C.F.}$ are always perpendicular), $dw/dt = \overrightarrow{W'} \cdot \overrightarrow{C.F.}$ Between points 1 and 2 the vector $\vec{r}$ has a negative value (assuming the constant direction of $\overrightarrow{W'}$ to be positive) so that the power being supplied to the bird by the air is

$$\frac{dw}{dt} = M \frac{Vc^2}{r} W' \sin\theta \quad (14)$$

along the semicircle 1 to 2. Physically this means that the bird, in order to turn in the direction of the wind, must impart momentum to the air in a direction opposite to the wind and the kinetic energy lost by this air is imparted to the bird to increase his kinetic energy and absolute velocity. To find the total work done by the centripetal force along the circuit 1 to 2 we integrate the power over the time required to cover the distance,

$$w_{1,2} = \int_{t_1}^{t_2} M \frac{Vc^2}{r} W' \sin\theta \, dt \quad (15)$$

Let $\theta = \omega t$ where $\omega$ is the constant angular velocity of rotation of the bird, $\omega = \frac{Vc}{r}$. Then

$$dt = \frac{d\theta}{\omega} = \frac{r}{Vc}\, d\theta$$

and

$$w_{1,2} = \int_0^{\pi} M \frac{Vc^2}{r} W' \frac{r}{Vc} \sin\theta \, d\theta = 2\,MVcW' \quad (16)$$

Thus we see that the gain in kinetic energy of the bird (or of a sailplane) between points 1 and 2 is equal to the energy extracted from the *air* by the working of the centripetal force on the craft. Hence there is no need for an altitude loss to supply this energy; in fact altitude has been gained between points 1 and 2 in the amount