THE THEORY OF SOARING FLIGHT IN VORTEX SHELLS - PT. 2

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(*Note: Last month, Part 1 of this paper contained an introduction and discussed the theory of bouyant convections.*)

AERODYNAMICS OF THE SPIRAL GLIDE

From the preceding description of thermal structure it is obviously necessary that a sailplane satisfy two basic requirements if it is to make efficient use of the upcurrents. First the radius of turn r must be sufficiently small that the craft remains in the region of strong vertical velocity. But it is also necessary when the craft has reached this radius that its sinking velocity $\dot{z}$ relative to the air has not increased so much that it is greater than the vertical air velocity in the thermal at that radius; otherwise, little has been accomplished. For maximum utilization, the conditions are even more stringent. As will be discussed in the next section, a sailplane can remain in continuous equilibrium flight within the thermal shell only if its sinking velocity relative to the air is equal to or less than the vertical velocity of the air *relative to the thermal core* at some radius. For analysis purposes it is therefore necessary to derive the expressions for the relation of the sinking velocity of the craft and the radius of turn.

Since weight is the only external force acting on a sailplane, all other forces are of aerodynamic origin and hence are determined only by the velocity of the craft relative to the air. If the form of the relative air motion is known, the motion of the craft relative to the earth is easily obtained by adding (vectorially) the velocity of the craft relative to the air with the velocity of the air relative to the earth. It is sufficient therefore for the determination of the aerodynamic effects on r and $\dot{z}$ to consider the mechanics of an equilibrium spiral glide in still air, which is necessarily the flight path of a circling sailplane.

Consider the sailplane in such an equilibrium glide with a radius of turn r; a force summation along and normal to the instantaneous flight direction gives

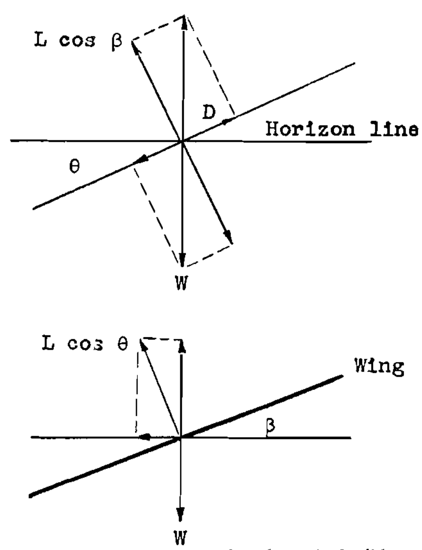

Fig. 4. Force systems for the spiral glide.

$$D = W \sin \theta \qquad (2)$$

$$L \cos \beta = W \cos \theta \qquad (3)$$

where D = drag, L = lift, β = bank angle, θ = flight path angle, and W = craft weight. Equating the aerodynamic centripetal force to the radial inertia force we obtain

$$L \sin \beta = \frac{W}{g} \frac{(V \cos \theta)^2}{r} \qquad (4)$$

where g = acceleration due to gravity, r = radius of turn, and V = aerodynamic flight velocity. The radius of turn r is obtained by solving equation (4) for r,

$$r = \frac{W}{S} \frac{2}{\rho g} \frac{\cos^2 \theta}{C_L \sin \beta} \qquad (5)$$

using the relation $L = C_L \frac{1}{2} \rho V^2 S$ to eliminate L. From the force relations it follows that

$$\cos \theta = \frac{C_L \cos \beta}{[C_D^2 + (C_L \cos \beta)^2]^{\frac{1}{2}}} \qquad (6)$$

so that r becomes

$$r = \qquad (7)$$

$$\frac{W}{S} \frac{2}{\rho g} \frac{C_L \cos^2 \beta}{\sin \beta \, [C_D^2 + (C_L \cos \beta)^2]}$$

This gives the relation between the radius of turn and the wing loading W/S, the altitude (ρ is a function of altitude), bank angle β, and angle of attack α (C_L and C_D are functions of α through the sailplane drag polar).

To obtain the sinking velocity $\dot{z}$, we note that $\dot{z} = V \sin \theta$. Then in the spiral glide

$$V = \left[\frac{L}{\frac{1}{2} \rho C_L S} \right]^{\frac{1}{2}} = \left[\frac{W \cos \theta}{\cos \beta \frac{1}{2} \rho C_L S} \right]^{\frac{1}{2}} \qquad (8)$$

using equation (3), and

$$z = \left(\frac{W}{S} \right)^{\frac{1}{2}} \left(\frac{2}{\rho} \right)^{\frac{1}{2}} \frac{\cos^{\frac{1}{2}} \theta \sin \theta}{(C_L \cos \beta)^{\frac{1}{2}}} \qquad (9)$$

From equation (6) and the relation

$$\sin \theta = \frac{C_D}{[C_D^2 + (C_L \cos \beta)^2]^{\frac{1}{2}}} \qquad (10)$$

we obtain finally

$$\dot{z} = \qquad (11)$$

$$\left(\frac{W}{S} \right)^{\frac{1}{2}} \left(\frac{2}{\rho} \right)^{\frac{1}{2}} \frac{C_D}{[C_D^2 + (C_L \cos \beta)^2]^{\frac{3}{4}}}$$

This relation gives the value of the sinking velocity of the craft relative to the air as a function of the various aerodynamic parameters.

The relation between r and $\dot{z}$ can be determined for any sailplane provided a calculated or experimental polar is available. From the polar, the parameter α (angle of attack) determines the simultaneous values of C_L and C_D. The wing loading is of course a constant and by specifying the bank angle β and altitude ρ, the

About the Author

Don Cone is an Aeronautical Research Engineer with NASA at the Langley Research Center in Virginia. He has authored numerous papers on various aerodynamic subjects ranging from subsonic wing theory and boundary layer control to hypersonic flow. His degrees include an Associate in Arts (Chemistry) at Armstrong College in Savannah, Georgia, a Bachelor of Chemical Engineering at the Georgia Institute of Technology, and a Master of Aeronautical Engineering at the University of Virginia. He is currently studying for a PhD in Aero-Space Engineering at the Virginia Polytechnic Institute. He is 29, married, and has two children. His soaring interests are primarily confined to theoretical and technical studies; however, he is also interested in the National and International Soaring Competitions, and in the subject of man-powered flight, and is presently attempting to form an active soaring club in Hampton, Va.