and so neglect C_D^2 in the denominators. This assumption is certainly valid for soaring planes. The equations then become

$$r = \frac{W}{S}\ \frac{2}{\rho g}\ \frac{\csc \beta}{C_L} \qquad (21)$$

$$\dot{z} = \left(\frac{W}{S}\right)^{1/2} \left(\frac{2}{\rho}\right)^{1/2} \sec^{1.5} \beta\ \frac{C_D}{C_L^{1.5}} \qquad (22)$$

Let us choose a value for β, say $\beta = 20°$. We do not want β too large (say not larger than 40°) since then $\dot{z}$ would become large because sec β appears to a power greater than unity. (Actually the optimum value of β can be determined by a detailed analysis.) Then for a given W/S we want C_L maximum large enough that r will be well within the thermal [see equation (21)]. In general, C_L will be large, especially if the model thermal is small. Considering equation (21) we see that $\dot{z}$ will be a minimum when $C_L^{1.5}/C_D$ is a maximum. Then clearly we want $C_L^{1.5}/C_D$ to be as large as possible, but just as important this maximum must occur at a large value of C_L so that r will be simultaneously small. This indicates two important requirements for thermal sailplane geometry.

First, since the induced drag increases as the square of the lift coefficient for conventional wings, a large *effective* aspect ratio is very necessary. Secondly, the parasite drag must be minimized at high C_L values. However, practical considerations and conflicts enter the picture and seriously complicate the matter. Consider aspect ratio first. A large geometric aspect ratio A requires a high wing structural weight, so that too large an A will increase W/S to a point where the A is actually detrimental. This fact has long been appreciated; however, in the present case it is much more critical since r is so sensitive to W/S. Thus a balance must be struck between W/S and A so as to get $\dot{z}$ and r within the desired range. Another factor becomes important when r is small. At high C_L values the aerodynamic velocity is low and the Reynolds numbers become small. A large aspect ratio means small chord values and it may well be that the chord Reynolds numbers become so low that laminar boundary layer flow will exist and will cause early wing stall to occur so that high C_L's cannot be achieved and small radii cannot be flown. Thus the thermal energy could not be touched. Large aspect ratio is also detrimental from the standpoint of parasite drag. The thick wing root sections necessary to house the deep spar have high profile drag at moderate C_L's and early separation, giving prohibitive form drag and seriously limiting maximum C_L. Another fault of large aspect ratio wings in tight turns is the induced drag increase over that of linear flight at equal C_L values. As the wing turns, the large span causes an appreciable variation of the velocity across the span, with a consequent asymmetrical loading. The strong aileron deflections necessary for trim accentuate the irregularity. The induced drag increase due to the asymmetrical loading may fully cancel the expected benefits of having high A: then we have also lost from the standpoint of minimum radius of turn. Finally, the asymmetrical lift loading just discussed gives rise to rolling moments of considerable magnitude with large span wings, far beyond the capability of any normal aileron or spoiler system to control. The result is that once the wing has become involved in a tight turn, a stall and spin is inevitable. The asymmetrical loading produces a large rolling moment which increases the bank angle which decreases the turn radius which increases the load asymmetry, etc. In trying to use small, weak thermals the parasite drag associated with the aileron control needed to balance the rolling moments is prohibitive. It is clear that very large aspect ratios are actually detrimental in the *maximum* utilization of thermal currents. A span of 50 to 60 feet is unthinkable for an efficient thermal soarer. Current sailplanes can use only the largest of thermals.

It thus appears that a limit may exist as to how closely man can approach the efficiency of birds in soaring. The limit is purely one of size. The size of the sailplane to support the necessary weight efficiently must be much larger than the largest soaring bird, so we cannot reasonably expect to use the smallest thermals available to birds because of the aspect ratio effects. However, there is a promising possibility of minimizing the size effect of aspect ratio. It is interesting to note that no land soaring birds have a large A. The average A is about 6. With this value they seem to have optimized all the various factors so efficiently that they can utilize all but the smallest thermals. However, the birds have an additional advantage. Their effective A appears to be much larger than the geometrical A. The secret is the slotted wing tip. The aerodynamics of this device are extremely complex but the problem has been practically solved. It should be mentioned that actual wind tunnel tests on the wing of a Black Vulture at very high values of C_L, just below the stall point ($C_L = 1.5$), showed that despite the low A, the trailing vorticity was amazingly small. (See the author's "On the Thermal Soaring of Birds," submitted for publication in the *Condor*, July, 1960.) Here perhaps lies the answer to our size problem with thermal soaring planes.

As concerns the parasite drag, the laminar flow wings so highly regarded in conventional sailplanes certainly have no place in thermal soaring, unless perhaps some means for triggering boundary layer turbulence is available for tight turns. The laminar flow leads to premature separation at high C_L values and is thus unacceptable for flying in small thermals of high intensity, the situation then being that we have a quite adequate power supply in the thermal if we can only get in to utilize it.

To summarize, the craft must be constructed such that the wing loading, effective aspect ratio, and parasite drag attain a proper balance to allow the use of a sufficiently large radius range where $\dot{z}(r)$ is well below $v(r)$, i.e., a large equilibrium region must be available. The working out of this balance successfully with present aerodynamic knowledge and structural skills depends critically on the properties of the "model" thermal, and hence on the particular geographical soaring region for which the craft is being designed.

We have so far considered the design of sailplanes based on the model, or most prevalent, thermal. The results of the brief analysis have indicated a few of the more important problems and have indicated some of the major design requirements. The nature of the model thermal dictates the design. For example, if the model thermal were large enough and intense enough we could soar nicely in the F-102. However, nature, though prolific, is ever economical and soaring experience by man has shown the thermals to be on the conservative side. On the other hand, birds have shown that they are not so conservative that proper design cannot successfully exploit them.

For *maximum* soaring efficiency we must be able to use the smallest, weakest thermals available. The require-