

# THE THEORY OF SOARING FLIGHT IN VORTEX SHELLS - PT. 3

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## ANALYSIS AND CONCLUSIONS

We have now covered the basic elements of the theory. Let us apply these results to the case of practical thermal soaring by man and see what conclusions can be drawn. The ultimate problem is to design a sailplane which will allow maximum utilization of the thermals occurring over a given region or locality. Let us assume for the moment that some how we have determined that of all the sizes and intensities of thermal shells formed over the region on a yearly or seasonal basis, the most frequent size is of radius  $R_1$ , circulation  $\Gamma_1$ , and core volume  $2\pi^2\alpha_1^2R_1$ , (volume of the vortex ring). By invoking the thermal theory, we can prepare a thermal diagram for this particular thermal, such as shown in figure 9. Now the sizes of thermals which form over the given area will no doubt vary from tiny puffs to enormous shells hundreds of yards in diameter, and the intensity will vary from shell to shell even for equal sizes. But the particular shell we have chosen, let's say our model thermal, appears to be sufficiently prevalent that it (and its larger neighbors) will make thermal soaring relatively certain provided the sailplane can attain equilibrium in it. The first thing we do with the diagram is to superpose on it the known (estimated or measured)  $\dot{z}$  vs.  $r$  relation (dashed curves of figure 9) for various existing sailplanes so we may see how well modern sailplanes meet the thermal

requirements. The figure tells us several things. First, if  $\dot{z}$  vs.  $r$  doesn't intersect any of the  $v$  vs.  $r$  curves for any value of  $\eta$  (such as curve A) it is obvious that the sailplane can never attain equilibrium in the thermal since  $\dot{z} > v$  for all radii. The altitude which *can* be gained in this case can be estimated for any constant radius by integrating the difference between  $\dot{z}$  and  $v$  as the sailplane sinks thru the shell. If  $\dot{z}(r)$  intersects  $v(r)$ , as shown by curve B, equilibrium within the shell *can* be attained and the sailplane will constantly rise with the shell with velocity  $V'$ . The intersection points of  $\dot{z}(r)$  with  $v(r)$  give the values of  $\eta$  at which the craft will fly in this region for a given value of  $r$ . Thus it is seen that an appreciable range of equilibrium radii may be available for a properly designed craft and as previously mentioned, no matter where in this region the craft flies it will climb with velocity  $V'$  relative to earth. There is no critical need in such cases to have precise radius control. The flight is automatic. Indeed it is necessary to maintain angle of attack and angle of bank only within rather wide limits. It is not absolutely necessary to be able to attain equilibrium over a *large* portion of the shell, but the advantage of a large region is that more room is allowed for control error. The ability to attain equilibrium over the entire shell is the principal reason why soaring birdflight appears so automatic and devoid of control motions. The

thermal diagram analysis will show us what we may expect from existing sailplanes for thermal flight over a given locality.

Suppose now, however, we want to design a craft which will be able to make *maximum* use of the model thermal. The basic requirement is obviously to make  $\dot{z}(r)$  as flat as possible and as small in value as possible, at least flat enough for the given thermal that an appreciable equilibrium area will exist. It should be noted that, in general, if a craft can use a certain size thermal, it will be able to use all larger thermals of similar strength since  $\dot{z}$  always decreases as  $r$  increases. Since equations (7) and (11) relate the shape of  $\dot{z}(r)$  to the aerodynamic properties of the craft, the design problem reduces to an analysis of these equations.

The first parameter (and probably the most important) is wing loading,  $W/S$ . From equation (7),  $r$  is directly proportional to  $W/S$ . From equation (11)  $\dot{z}$  is directly proportional to  $(W/S)^{1/2}$ , so lowering  $W/S$  not only allows us to get into the stronger upcurrent region near the center of the thermal, but also reduces the value of  $\dot{z}$ . If we set a maximum value on  $C_L$  so as to prevent stall, say  $C_L \leq 1.5$ , the radius of turn is limited by  $\beta$ . But a large  $\beta$  means a high sinking velocity from equation (11) so  $\beta$  must be limited to a small value. Therefore the minimum usable radius is set by  $W/S$ . It is evident that from the standpoint of *thermal* soaring the lowest possible wing loading is desirable. The lower the wing loading, the smaller the circle the craft can fly and the smaller the thermals it can utilize. Since  $\dot{z}$  also decreases with wing loading the weaker may be the thermals that can be utilized. Since the wing loadings of many modern sailplanes are 5 to 7 times those of the heavier soaring birds, it is clear why the birds are so much superior in thermal utilization. If they can use thermals down to 1/7 the size of those available to modern sailplanes, imagine the great increase in the number of total thermals available to the birds and hence the increase in certainty of sustained flight.

The second factor of importance is the relationship between  $C_L$  and  $C_D$ ; in other words the drag polar. As they stand in the exact equations (7) and (11), the factors containing  $C_L$  and  $C_D$  are somewhat involved. However, they are easily simplified. For the purposes of our analysis, in both equations assume  $C_D^2 \ll (C_L \cos \beta)^2$

Fig. 9. Illustration of the use of  $dZ/dt$  ( $r$ ) with the thermal diagram. Note:  $dZ/dt = \dot{z}$ .

