ture is quite similar to that of the cylinder vortex wake generated by a lifting airfoil.* A large part of the energy required for the formation of a vortex ring is associated with the flow in the shell exterior to the ring itself. In this way the concentrated thermal energy of the initial protuberance is used to impart a small velocity to a very large mass of air, a form much more useful for soaring flight. Figure 2 is a cross-sectional view of the flow within the moving thermal shell. The streamlines are all closed. That is, the fluid within the shell continuously circulates around the vortex ring while the ring rises through the atmosphere with a vertical velocity V'. The fluid once entrapped always remains within the shell and travels with the vortex. The velocity with which the ring (and its accompanying shell) rises is given by

$$V' = \frac{\Gamma}{4\pi R}\left[\ln\frac{8R}{a} - \frac{1}{4}\right] \quad (1)$$

where Γ is the vortex ring circulation, R is the ring radius, and a is the core radius.

The buoyancy of the vortex ring has a pronouced effect on the overall motion. Since the core has a buoyant force acting as it rises, work is being done on the air and the potential energy released must be used to increase the impulse of the vortex ring. If we assume that the circulation of the ring remains essentially constant after development, the increase in the impulse requires that the diameter of the ring increase as the ring rises. Since the ring diameter determines the velocity V' of the ring (equation 1), the ring will in general lose velocity as it rises. A rather simple analysis shows that, strangely enough, the greater the buoyancy of the ring the faster it will decrease its vertical velocity as it rises. Hence the relatively low buoyancies of natural thermals appear conducive to fast rising thermal shells. Additionally, the lower buoyancy rings will rise to much greater altitudes. The increasing diameter of the ring also increases the size of the external shell so that the size of a thermal shell (vortex ring and accompanying body of fluid) increases appreciably with altitude; a given thermal at say 5,000 feet altitude is many times larger than it was at 500 feet. This fact is very important to the larger soaring birds. Hankin* noted that such birds consistently soared at much higher altitudes than the smaller birds, but was unable to offer a satisfactory explanation. This topic is covered in detail in the author's "On the Thermal Soaring of Birds" (op.cit.).

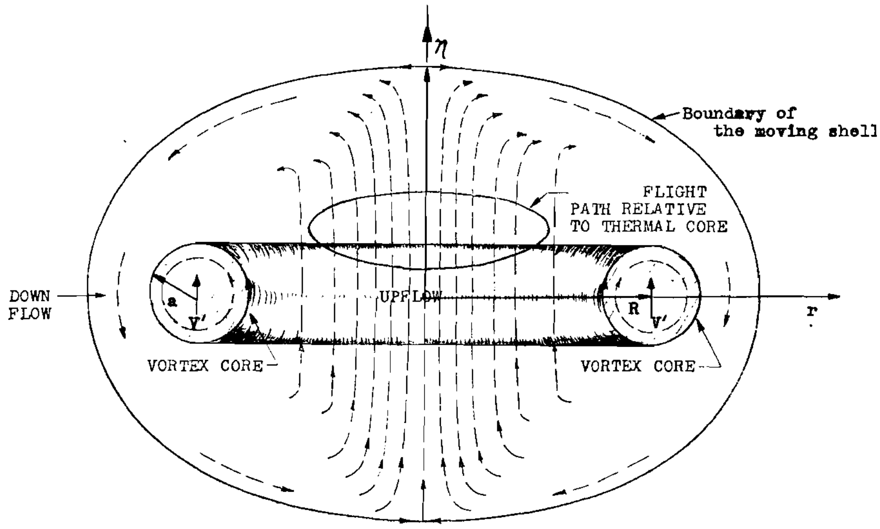

Fig. 2. General presentation of the flow relative to the vortex core in a thermal shell. (Cross-sectional view except for perspective of circling flight path.)

The entire velocity field associated with a vortex ring of radius R and core radius a can be calculated provided the circulation Γ is known. The calculation is, however, quite complex. The theory of buoyant vortex rings allows us to calculate the value of R as a function of altitude. Then assuming quasi-steady flow at any altitude, it is possible to quantitatively predict the entire velocity field within the thermal shell at any time and also to plot the path of the thermal relative to the earth. This information allows us to establish quantitatively the properties a sailplane must possess in order to make maximum use of the thermal energy. We now consider the details of the motion of the thermal shell.

At altitude h the vortex ring has grown to radius R and core radius a so that the ring (and the entire shell) is rising vertically relative to the earth with the velocity V'. Inside the shell the air is circulating in closed paths around the vortex core. The mathematical description of the velocity field is somewhat complex and is given in detail by H. Lamb.* The picture of the streamlines of flow relative to the core is quite simple, however, as shown in another work.** In general, the shape of the fluid body which accompanies the ring depends very much on the values of R and a for a given Γ. For values of R/a less than 10 the shape of the shell is oval, somewhat as shown in figure 2. For a larger value of R/a the oval develops a depression in the middle of the top and bottom surfaces. This depression increases until finally at $R/a = 86$ the shell has assumed a ring form. The nature of formation of free atmospheric thermals is such that we may always expect R/a to be a small number (say less than 10).

For soaring applications we are interested primarily in the vertical components of the velocities *relative to the thermal core*, as will be evident later. Lateral velocity components exist near the boundaries of the shell, but they are small and require no consideration for our immediate purposes. The nature of the vertical velocity distribution relative to the thermal core is illustrated in figure 3, which was prepared from experimental data given by J. R. Scorer.*** The values as experimentally determined are quite similar to those predicted by vortex ring theory for a value of $R/a \rightarrow 5$.

* C. D. Cone, Jr.: "Theoretical Investigation of the Vortex Sheet Deformation Behind a Highly Loaded Wing and Its Effect on the Lift," *N.A.S.A. Technical Note D*-657, Dec., 1960, Appendix A.

* E. H. Hankin (op.cit.).

* H. Lamb: *Hydrodynamics*, Dover Publications, New York, 1945, p. 236-241.

** L. Prandtland O.G. Tietjens: *Fundamentals of Hydro-and Aeromechanics*, *Dover Publications*, New York, 1957, p. 213.

*** J. R. Scorer: *Natural Aerodynamics*, Pergamon Press, New York, 1958.